



Corrigendum to “Topological entropy for impulsive differential equations” [*Electron. J. Qual. Theory Differ. Equ.* 2020, No. 68, 1–15]

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Abstract. The aim of this corrigendum is two-fold: (i) to indicate the incorrect parts in two propositions of our recent paper with the same title, (ii) to state the correct statements.

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1 Incorrect propositions, their consequences and corrections

The vector impulsive differential equation under our consideration in [1] takes the form

$$\begin{cases} x' = F(t, x), & t \neq t_j := j\omega, \text{ for some given } \omega > 0, \\ x(t_j^+) = I(x(t_j^-)), & j \in \mathbb{Z}, \end{cases} \quad (1.1)$$


where $F: \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the Carathéodory mapping such that $F(t, x) \equiv F(t + \omega, x)$, equation $x' = F(t, x)$ satisfies a uniqueness condition and a global existence of all its solutions on $(-\infty, \infty)$. Let, furthermore, $I: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be a compact continuous impulsive mapping such that $K_0 := \overline{I(\mathbb{R}^n)}$ and $I(K_0) = K_0$.

Unfortunately, there is a gap in the second part of the proof of the following proposition.

Proposition 1.1 (cf. [1, Proposition 3.1]). *Let $T_\omega: \mathbb{R}^n \rightarrow \mathbb{R}^n$ be the associated Poincaré translation operator along the trajectories of $x' = F(t, x)$, such that $K_1 := T_\omega(K_0)$ and $K_0 \subset K_1$. Then the equality*

$$h(I|_{K_1} \circ T_\omega|_{K_0}) = h(I|_{K_0}) \quad (1.2)$$

holds for the topological entropies h of the maps $I|_{K_1} \circ T_\omega|_{K_0}: K_0 \rightarrow K_0$ and $I|_{K_0}: K_0 \rightarrow K_0$.

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Since the equality (1.2) was used in the proof of the first main theorem (see [1, Theorem 3.5]), this theorem can be corrected in the simplest way, when assuming (1.2) or, more generally the inequality

$$h(T_\omega|_{K_0} \circ I|_{K_1}) \geq h(I|_{K_0}), \quad (1.3)$$

explicitly. Then the following correction has rather a character of a proposition.

Theorem 1.2. *The vector impulsive differential equation (1.1) exhibits under (1.3) chaos in the sense of a positive topological entropy of the composition $I|_{K_1} \circ T_\omega|_{K_0}$, i.e. $h(I|_{K_1} \circ T_\omega|_{K_0}) > 0$, provided $I(K_0) = K_0$ and $K_0 \subset K_1$, where $K_0 := \overline{I(\mathbb{R}^n)}$ and $K_1 := T_\omega(K_0)$, jointly with $h(I|_{K_0}) > 0$.*

Despite this gap, all the related illustrative examples (see [1, Examples 3.7–3.9]) can be shown to be correct, when verifying (1.3), by means of e.g. a slightly generalized version of [2, Proposition 3.2].

The same type of a gap is in the proposition for the problem (1.1) considered, under the natural additional assumptions

$$F(t, \dots, x_j, \dots) \equiv F(t, \dots, x_{j+1}, \dots), \quad j = 1, \dots, n, \quad (1.4)$$

and

$$I(\dots, x_j, \dots) \equiv I(\dots, x_{j+1}, \dots) \pmod{1}, \quad j = 1, \dots, n, \quad (1.5)$$

on the torus $\mathbb{R}^n/\mathbb{Z}^n$ (see [1, Proposition 4.1]). Quite analogously, the second main theorem (see [1, Theorem 4.3]) can be corrected by the additional technical assumption

$$h((\tau \circ T_\omega) \circ (\tau \circ I)) \geq h(\tau \circ I), \quad (1.6)$$

where $\tau: \mathbb{R}^n \rightarrow \mathbb{R}^n/\mathbb{Z}^n$ denotes the natural projection.

Since on tori, we have to our disposal the Ivanov inequality for the lower estimate of topological entropy in terms of the asymptotic Nielsen numbers (see [4] and cf. [1, Proposition 2.7]), the third main theorem in [1, Theorem 4.6] remains valid, even without verifying (1.6), in the following way.

Theorem 1.3. *Consider, under the above assumptions and (1.4), (1.5), the vector impulsive differential equation (1.1) on $\mathbb{R}^n/\mathbb{Z}^n$. Assume that the impulsive mapping $(\tau \circ I): \mathbb{R}^n/\mathbb{Z}^n \rightarrow \mathbb{R}^n/\mathbb{Z}^n$ is homotopic to a continuous map $f: \mathbb{R}^n/\mathbb{Z}^n \rightarrow \mathbb{R}^n/\mathbb{Z}^n$ such that $N^\infty(f) > 1$, i.e.*

$$\limsup_{m \rightarrow \infty} |\lambda(f^m)|^{\frac{1}{m}} > 1,$$

where $\lambda(f^m)$ stands for the Lefschetz number of the m -th iterate of f .

Then

$$\begin{aligned} h((\tau \circ I) \circ (\tau \circ T_\omega)) &\geq \limsup_{m \rightarrow \infty} \frac{1}{m} \log N\left((\tau \circ I) \circ (\tau \circ T_\omega)^m\right) \\ &= \limsup_{m \rightarrow \infty} \frac{1}{m} \log N((\tau \circ I)^m) = \limsup_{m \rightarrow \infty} \frac{1}{m} \log N(f^m) > 0 \end{aligned}$$

holds, where $N(f^m)$ denotes the Nielsen number of the m -th iterate of f , and subsequently equation (1.1) exhibits on $\mathbb{R}^n/\mathbb{Z}^n$ chaos in the sense of a positive topological entropy of the composition $(\tau \circ I) \circ (\tau \circ T_\omega)$.

That is also why that all the related illustrative examples (see [1, Examples 4.5, 4.7, 4.9]) remain on this basis correct.

2 Concluding remarks

To verify the inequalities (1.3) and (1.6) is not an easy task (see e.g. [3]). We will try to affirm them at least in some particular cases elsewhere. In \mathbb{R} , the most promising way seems to be via the statements along the lines of [2, Proposition 3.2].

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